Calculus 1: Summary

By Jirawut Thongraar

Last updated on: 18 August 2021

Permission to use, copy, modify, and/or distribute this document for any purpose without fee is hereby granted, provided that this notice appears in all copies. This document is provided “as is” and the author makes no guarantees regarding the validity, correctness, and completeness of the information provided. In no event shall the author be liable for any direct, indirect, or consequential damages, or any damages whatsoever arising out of or in connection with the information provided by this document.

# Limits

Limit exists if and only if the approach from left and right is the same:

## Properties of Limits

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

## Calculating for Limits

### Direct Substitution

If is polynomial or rational function or is continuous at *a*, then .

### Limit via Similar Function

If except at , then .

If for is near (possibly except at ), then .

### Squeeze Theorem or Sandwich Theorem

If when *x* is near *a* (possibly except at ), then

## Applications

### Asymptotes

Vertical: if

Horizontal: if

Tip: For rational function, you only need to consider the largest degree term as it is the most significant.

### Continuous

If *f* is continuous at , then .

If and is continuous, then is also continuous.

If is continuous at and is continuous at , then is continuous at .

### Intermediate Value Theorem

If is continuous at , then there exists such that is between and .

# Derivatives

## Properties of Derivatives

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Intermediate Form and L’Hospital’s Rule

If it is an intermediate form of or

This can only be used on the above intermediate form, but some form can be transformed:

# Minimum and Maximum

Global Maximum and Minimum is where the function is the greatest or the least in all of domain.

Local Maximum and Minimum is where the function is the greatest or the least near some points.

The concept is intuitive, although domain endpoints cannot be local maximum and minimum.

## Critical Number

Point where or does not exist.

## Closed Interval Theorem for Global Maximum or Minimum

Global Maximum or Minimum of in range , is the maximum or minimum of

* Value of at and .
* Value of at all critical points in .

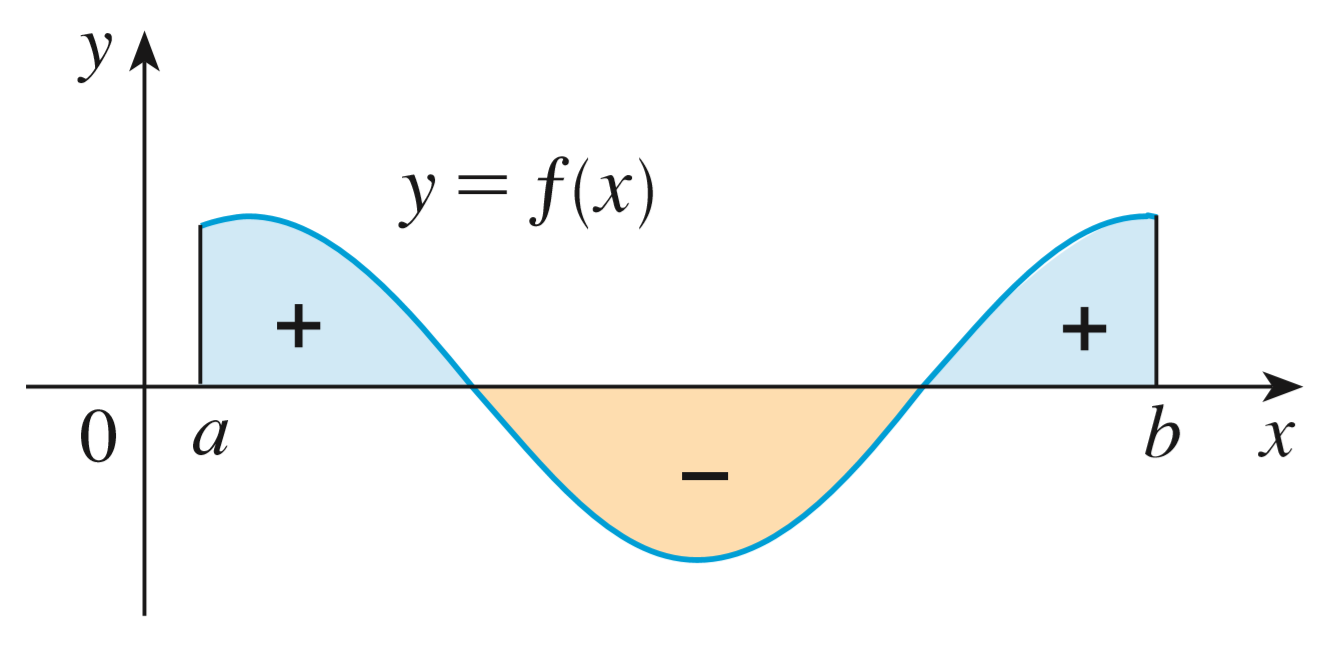
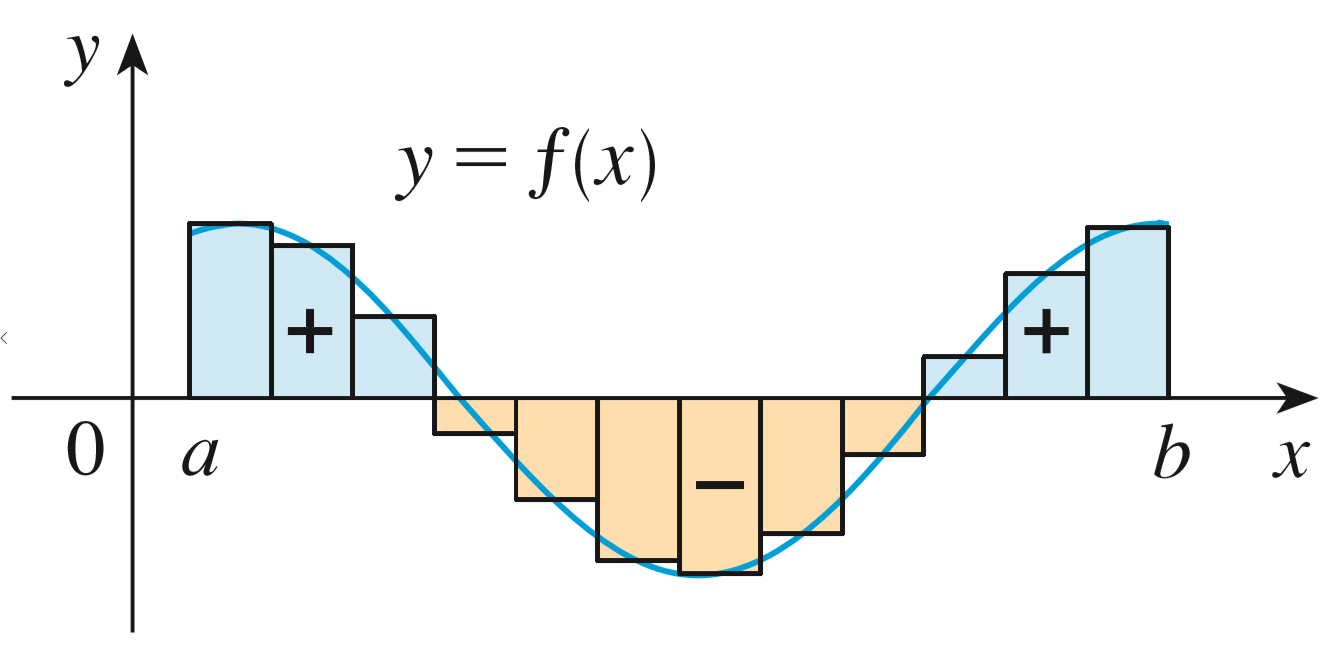
## Derivative Test for Local Extreme Values

Let be a critical number of continuous function .

* If changes from positive to negative, then is the local maximum.
* If changes from negative to positive, then is the local minimum.
* If and , then is the local maximum.
* If and , then is the local minimum.

# Integrals

Integral is a way to find the area under the curve. It is the limit of the approximation of the area by dividing it into tiny strips. The area may be negative if the curve is below the -axis.



## Properties of Integrals

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

## Calculating for Integrals

### Squeeze Theorem

If for , then

### Antiderivative

Useful for finding integral of most simple functions, such as .

|  |  |
| --- | --- |
|  | is “antiderivative” of . |
|  | Integral is the subtraction of antiderivative function. |
|  | Derivative and Integral is inverse of each other. |

### Substitution

Useful for finding integral of composite functions.

Let

### Symmetries

If is even (y-axis symmetry), then .

If is odd ( symmetry), then .

### Integration by parts

Useful for finding integral of multiplication products.

# Sequences and Series

A sequence is an ordered set of values:

A series is the sum of the first terms of the sequence:

A sequence/series “converges” if limit approach infinity exists (otherwise it “diverges”):

A series will diverge if its sequence doesn’t converge to 0:

To calculate the limit of a sequence or series, it may be useful to extend it into real numbers:

Let be a function where for (Don’t forget this step!)

### Squeeze Theorem for Sequences

If , then .

The absolute function is useful for this: .

## Arithmetic Sequence and Series

A sequence with a constant increase/decrease. Such as

Arithmetic Sequence and Series **always diverges** (if ).

## Geometric Sequence and Series

A sequence with a constant ratio. Such as

Geometric Sequence and Series **converges if** .

Proof of Geometric Series Formula:

## Faulhaber's formula

(when )

Calculating Formula:

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
|  |  |  |
|  |  |  |

Add it all together:

Solve for .

## Harmonic Series

Harmonic Series **always diverges**.